Reading: Sections 6.12 and 6.13 in Chapter 6 of textbook.
In-class quiz: Tuesday, November 7, 2017

Problem 1. Problem 6.62

Solution. We solve the following optimization problem:

$$\min_{P_1, P_2} C_1(P_1) + C_2(P_2)$$

s.t. $1000 - P_1 - P_2 = 0$.

The Lagrangian is

$$\mathcal{L}(P_1, P_2, \mu) = C_1(P_1) + C_2(P_2) + \mu(1000 - P_1 - P_2)$$

$$= 600 + 18P_1 + 0.04P_1^2 + 700 + 20P_2 + 0.03P_2^2 + \mu(1000 - P_1 - P_2).$$

At the optimal point, the following are satisfied:

$$\frac{\partial \mathcal{L}}{\partial P_1} = 18 + 0.08P_1 - \mu = 0$$

$$\frac{\partial \mathcal{L}}{\partial P_2} = 20 + 0.06P_2 - \mu = 0$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = 1000 - P_1 - P_2 = 0.$$

Solving the above, we obtain $P_1 = 442.86$ MW, $P_2 = 557.14$ MW, and $\mu = 53.43$/MWh at the optimal point. This leads to a total cost of

$$C_T = 600 + 18P_1 + 0.04P_1^2 + 700 + 20P_2 + 0.03P_2^2 = 37571/\text{hr}$$

Problem 2. Problem 6.63

Solution. Since the unlimited solution violates the $P_2$ limit constraint, set $P_2 = 400$ MW, which means $P_1 = 600$ MW. The incremental cost is determined by the unit that is not at its limit, i.e.,

$$\mu = \frac{dC_1}{dP_1} = 18 + 0.08P_1 = 18 + 0.08(600) = 66$/MWh.

The total cost is

$$C_T = 600 + 18P_1 + 0.04P_1^2 + 700 + 20P_2 + 0.03P_2^2 = 39300/\text{hr}$$
Problem 3. Problem 6.64

Solution. At the optimal point, the following are satisfied:

\[
\frac{\partial C_1}{\partial P_1} = 18 + 0.08P_1 = \mu \\
0.95\frac{\partial C_2}{\partial P_2} = 0.95(20 + 0.06P_2) = \mu \\
P_1 + P_2 = 1000.
\]

Solving the above, we obtain \( P_1 = 423.35 \) MW, \( P_2 = 576.64 \) MW, and \( \mu = $51.87/MWh \) at the optimal point. This leads to a total cost of

\[
C_T = 600 + 18P_1 + 0.04P_1^2 + 700 + 20P_2 + 0.03P_2^2 = $37598/hr
\]

Problem 4. Problem 6.67

Solution. In this problem, we consider three regions of operation:

(1) At minimum load with \( P_L = 55 \) MW, both generators must be dispatched at minimum capacity, i.e., \( P_1 = 25 \) MW and \( P_2 = 30 \) MW. Furthermore, the incremental costs of the two generators are

\[
\lambda_1 = \frac{dC_1}{dP_1} = 0.02P_1 + 2 = 0.02(25) + 2 = $2.5/MW, \\
\lambda_2 = \frac{dC_2}{dP_2} = 0.008P_2 + 2.6 = 0.008(30) + 2.6 = $2.84/MW.
\]

Since \( G_1 \) is cheaper with \( \lambda_1 = $2.5/MW \), we dispatch \( G_1 \) until \( \lambda_1 = \lambda_2 \).

\[
2.84 = 0.02P_1 + 2 \implies P_1 = 42 \text{ MW}
\]

With generation levels of \( P_1 = 42 \) MW and \( P_2 = 30 \) MW, the load served is \( P_L = 72 \) MW.

(2) At maximum load with \( P_L = 350 \) MW, both generators must be dispatched at maximum capacity, i.e., \( P_1 = 150 \) MW and \( P_2 = 200 \) MW. Furthermore, the incremental costs of the two generators are

\[
\lambda_1 = \frac{dC_1}{dP_1} = 0.02P_1 + 2 = 0.02(150) + 2 = $5/MW, \\
\lambda_2 = \frac{dC_2}{dP_2} = 0.008P_2 + 2.6 = 0.008(200) + 2.6 = $4.2/MW.
\]

Since \( G_1 \) is more expensive with \( \lambda_1 = $5/MW \), we can achieve lower total cost if we pull back \( G_1 \) generation. Hence, we reduce \( P_1 \) until \( \lambda_1 = \lambda_2 \).

\[
4.2 = 0.02P_1 + 2 \implies P_1 = 110 \text{ MW}
\]

With generation levels of \( P_1 = 110 \) MW and \( P_2 = 200 \) MW, the load served is \( P_L = 310 \) MW.

(3) For \( 72 \text{ MW} \leq P_L \leq 310 \text{ MW} \), \( \lambda_1 = \lambda_2 = \lambda \). In this range, we solve the following optimization problem:

\[
\min_{P_1, P_2} C_1(P_1) + C_2(P_2) \\
\text{s.t. } P_1 + P_2 - P_L = 0.
\]

The Lagrangian is

\[
\mathcal{L}(P_1, P_2, \mu) = C_1(P_1) + C_2(P_2) - \lambda(P_1 + P_2 - P_L) \\
= 100 + 2P_1 + 0.01P_1^2 + 80 + 2.6P_2 + 0.004P_2^2 + \lambda(P_L - P_1 - P_2).
\]
At the optimal point, the following are satisfied:

\[
\frac{\partial L}{\partial P_1} = 2 + 0.02P_1 - \lambda = 0
\]
\[
\frac{\partial L}{\partial P_2} = 2.6 + 0.008P_2 - \lambda = 0
\]
\[
\frac{\partial L}{\partial \lambda} = P_L - P_1 - P_2 = 0.
\]

Solving the above for \( P_L = 282 \) MW, we obtain \( P_1 = 102 \) MW, \( P_2 = 180 \) MW, and \( \lambda = $4.04/MWh \) at the optimal point.

Table 1: Problem 4 Summary

<table>
<thead>
<tr>
<th>( P_L ) [MW]</th>
<th>( P_1 ) [MW]</th>
<th>( P_2 ) [MW]</th>
<th>( \lambda_1 ) [$/MW]</th>
<th>( \lambda_2 ) [$/MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>([55, 72])</td>
<td>([25, 42])</td>
<td>30</td>
<td>0.02( P_1 ) + 2</td>
<td>2.84</td>
</tr>
<tr>
<td>((72, 310))</td>
<td>((42, 110))</td>
<td>((30, 200))</td>
<td>0.02( P_1 ) + 2</td>
<td>0.008( P_2 ) + 2.6</td>
</tr>
<tr>
<td>([310, 350])</td>
<td>([110, 150])</td>
<td>200</td>
<td>0.02( P_1 ) + 2</td>
<td>4.2</td>
</tr>
</tbody>
</table>