In-class quiz: Thursday, September 14, 2017

**Problem 1.** A three-phase line, which has an impedance of \((2 + j4) \, \Omega\) per phase, feeds two balanced three-phase loads that are connected in parallel. One of the loads is Y-connected with an impedance of \((30 + j40) \, \Omega\) per phase, and the other is \(\Delta\)-connected with an impedance of \((60 - j45) \, \Omega\) per phase. The line is energized at the sending end from a 60-Hz, three-phase, balanced voltage source of \(120\sqrt{3} \, \text{V (rms, line-to-line)}\). Determine:

1. The current, real power, and reactive power delivered by the sending-end source.

The per-phase total impedances of the line and loads is

\[
\bar{Z} = \bar{Z}_{\text{line}} + \bar{Z}_1|\bar{Z}_2| = 2 + j4 + \left( \frac{1}{30 + j40} + \frac{1}{20 - j15} \right)^{-1} = 24\angle0^\circ \, \Omega.
\]

Then, the source current is

\[
\bar{I}_s = \frac{\bar{V}}{\bar{Z}} = \frac{120\angle0^\circ}{24\angle0^\circ} = 5\angle0^\circ \, \text{A}.
\]

The complex power delivered by the source is

\[
\bar{S}_s = 3\bar{V}\bar{I}^* = 3(120\angle0^\circ)(5\angle0^\circ) = 1800\angle0^\circ \, \text{VA},
\]

with \(P_s = 1800 \, \text{W}\) and \(Q_s = 0 \, \text{MVar}\).

2. The line-to-line voltage at the load.

The phase voltage at the load is

\[
\bar{V}_L = \bar{V}_s - \bar{Z}_{\text{line}}\bar{I}_s = 120\angle0^\circ - (2 + j4)(5\angle0^\circ) = 110 - j20 \, \text{V} = 111.80\angle -10.3^\circ \, \text{V}.
\]

Therefore the line-to-line voltage at the load is

\[
\bar{V}_{L,L-L} = \sqrt{3}\bar{V}_L\angle30^\circ = \sqrt{3}111.80\angle -10.3^\circ + 30^\circ = 193.65\angle19.7^\circ \, \text{V}
\]

3. The current per phase in each load.

The per-phase current through the Y-connected load is

\[
\bar{I}_1 = \frac{\bar{V}_L}{\bar{Z}_1} = \frac{111.80\angle -10.3^\circ}{30 + j40} = 2.236\angle -63.4^\circ \, \text{A}.
\]

The per-phase current through the Y-connected equivalent of the \(\Delta\)-connected load is

\[
\bar{I}_{2,\phi} = \frac{\bar{V}_L}{\bar{Z}_2} = \frac{111.80\angle -10.3^\circ}{20 - j15} = 4.472\angle 26.57^\circ \, \text{A}.
\]

So the per-phase current of the \(\Delta\)-connected load is

\[
\bar{I}_{2,\Delta} = \frac{\bar{I}_{2,\phi}}{\sqrt{3}}\angle30^\circ = \frac{4.472}{\sqrt{3}}\angle26.57^\circ + 30^\circ = 2.582\angle56.57^\circ \, \text{A}.
\]
4. The total three-phase real and reactive powers absorbed by each load and by the line.

The 3φ complex power absorbed by the Y-connected load is

\[ S_1 = 3\bar{V}_L\bar{I}_1 = 3(111.80\angle -10.3^\circ)(2.236\angle -63.4^\circ)^* = 450.3 + j599.7 \text{ VA}. \]

The 3φ complex power absorbed by the Δ-connected load is

\[ S_2 = 3\bar{V}_{L,L-L}\bar{I}_{2,\Delta} = 3(193.65\angle 19.7^\circ)(2.582\angle 56.57^\circ)^* = 1200 - j900 \text{ VA}. \]

The complex power absorbed by the line impedance is

\[ \bar{S}_{\text{line}} = 3\bar{Z}_{\text{line}}\bar{I}_L^2 = 3(2 + j4)5^2 = 150 + j300 \text{ VA}. \]

The sum of the three quantities above is 1800 + j0 VA, which matches the value obtained in Part 1.

Check that the total three-phase complex power delivered by the source equals the total three-phase power absorbed by the line and loads.

**Problem 2.** Two three-phase generators supply a three-phase load through separate three-phase lines. The load absorbs 30 kW at 0.8 power factor lagging. The line impedance is (1.4 + j1.6) Ω per phase between generator G1 and the load, and (0.8 + j1) Ω per phase between generator G2 and the load. If generator G1 supplies 15 kW at 0.8 power factor lagging, with a terminal voltage of 460 V line-to-line, determine:

1. The voltage at the load terminals.
   
   The complex power supplied by G1 is
   
   \[ S_{1,3φ} = \frac{15}{0.8} \angle \cos^{-1} 0.8 = 18.75\angle 36.87^\circ \text{ kVA}. \]

   The current supplied by G1 is
   
   \[ \bar{I}_{G1} = \left[ \frac{S_{1,3φ}}{3\bar{V}_{G1}} \right]^* = \left[ \frac{18.75\angle 36.87^\circ}{3 \left( \frac{460}{\sqrt{3}} \right) \angle 0^\circ} \right]^* = 23.53\angle -36.87^\circ \text{ A}. \]

   So the phase voltage at the load is
   
   \[ \bar{V}_L = \bar{V}_{G1} - \bar{Z}_l \bar{I}_{G1} = \left( \frac{460}{\sqrt{3}} \right) \angle 0^\circ - (1.4 + j1.6)(23.53\angle -36.87^\circ) = 216.9\angle -2.74^\circ \text{ V}, \]

   and the line-to-line voltage at the load is
   
   \[ \bar{V}_{L,L-L} = \sqrt{3}(216.9)\angle 27.26^\circ \text{ V}, \]

2. The voltage at the terminals of generator G2.

   The complex power absorbed by the load is
   
   \[ S_{L,3φ} = \frac{30}{0.8} \angle \cos^{-1} 0.8 = 37.5\angle 36.87^\circ \text{ kVA}. \]

   The current into the load is
   
   \[ \bar{I}_L = \left[ \frac{S_{L,3φ}}{3\bar{V}_L} \right]^* = \left[ \frac{37.5\angle 36.87^\circ}{3(261.9\angle -2.74^\circ)} \right]^* = 57.63\angle -39.61^\circ \text{ A}. \]

   The current into the load is the sum of the currents supplied by the two generators, so
   
   \[ \bar{I}_{G2} = \bar{I}_L - \bar{I}_{G1} = 57.63\angle -39.61^\circ - 23.53\angle -36.87^\circ = 34.15\angle -41.5^\circ \text{ A}. \]

   And finally the voltage at the terminal of G2 is the sum of the voltage drop across the load and \( \bar{Z}_{l_2} \):
   
   \[ \bar{V}_{G2} = \bar{V}_L + \bar{Z}_{l_2} \bar{I}_{G2} = 216.9\angle -2.74^\circ + (0.8 + j1)(34.15\angle -41.5^\circ) = 259.8\angle -0.638^\circ \text{ V}. \]

And the line-to-line voltage at the terminal of G2 is

\[ \bar{V}_{G2,L-L} = \sqrt{3}(259.8)\angle 29.36^\circ \text{ V}. \]
3. The real and reactive power supplied by generator G2. The complex power supplied by G2 is

\[
\tilde{S}_{G2,3\phi} = 3\tilde{V}_{G2}\tilde{I}_{G2} = 3(259.8^\circ - 0.638^\circ)(34.15^\circ - 41.5^\circ) = 20.1 + j17.4 \text{kVA}.
\]

Hence, the real power supplied is \(P_{G2,3\phi} = 20.1 \text{ kW}\) and the reactive power supplied is \(Q_{G2,3\phi} = 17.4 \text{ kVar}\).

**Problem 3.** An unbalanced three-phase, Y-connected power system is shown in the figure below. The three phases have voltages \(V_a = 100\angle 0^\circ \text{ V}\), \(V_b = 100\angle -120^\circ \text{ V}\), \(V_c = 100\angle 120^\circ \text{ V}\). The impedances of loads A, B, C are \(Z_a = 10 \Omega\), \(Z_b = -j10 \Omega\), \(Z_c = j10 \Omega\).

1. What are the currents of each phase \(I_a, I_b, I_c\)?

\[
I_a = \frac{V_a}{Z_a} = \frac{100\angle 0^\circ}{10} = 10\angle 0^\circ \text{ A},
I_b = \frac{V_b}{Z_b} = \frac{100\angle -120^\circ}{-j10} = 10\angle -30^\circ \text{ A},
I_c = \frac{V_c}{Z_c} = \frac{100\angle 120^\circ}{j10} = 10\angle 30^\circ \text{ A}
\]

2. What is the current on the neutral line \(I_n\)?

\[
I_n = I_a + I_b + I_c = 10\angle 0^\circ + 10\angle -30^\circ + 10\angle 30^\circ = (10 + 10\sqrt{3})\angle 0^\circ = 27.3\angle 0^\circ \text{ A}
\]

3. What are the line voltages \(V_{ab}, V_{bc}, V_{ca}\)?

\[
V_{ab} = 100\sqrt{3}\angle 30^\circ = 173\angle 30^\circ \text{ V},
V_{bc} = 100\sqrt{3}\angle -90^\circ = 173\angle -90^\circ \text{ V},
V_{ca} = 100\sqrt{3}\angle 150^\circ = 173\angle 150^\circ \text{ V}
\]

4. Provide the phasor diagram of the phasors including the phase voltages, line voltages, phase currents and the current on the neutral line.

![Figure 1: phasor diagram](image)