Reading: Sections 11.1 – 11.3 in Chapter 11 of textbook.
In-class quiz: Thursday, November 30, 2017.

Problem 1. Given a turbine-generator unit rated 100 MVA with a per unit inertia constant, $H = 5$ sec.

(a) Calculate the kinetic energy stored at synchronous speed (3600 rpm).

(b) Compare the answer from (a) with the kinetic energy of a 10-ton truck going at 60 mph.

(c) Suppose that the generator is delivering 100 MW and then at $t = 0$, the line circuit breakers open. Calculate the shaft acceleration in rad/sec$^2$.

(d) At this rate, how long does it take for $\delta$ to increase from $\delta^0$ to $\delta^0 + 2\pi$ radians.

Problem 2. Consider the spring-mass system shown in Fig. 1.

\[ m \frac{d^2x}{dt^2} + a \frac{dx}{dt} + \sin x = f(t) \]

Figure 1: Spring-mass system for Problem 2.

(a) For $t < 0$, $f(t) = 0.5$. Find the steady-state $x^0$.

(b) For $0 \leq t$, $f(t) = 0.5 + \Delta f(t)$, where $\Delta f(t)$ is small. Find a linear differential equation that describes $\Delta x = x - x^0$ in terms of $\Delta f(t)$.

(c) Find the natural frequencies of the linearized system found in part (b) with $m = 1.0$ and $a = 0.01$. Suppose that $\Delta f(t) = 0.1u(t)$, where $u(t)$ is the unit step function. Roughly sketch $x(t)$ for $t \geq 0$, showing the initial value, final value, frequency, and decay of the lightly damped oscillatory component.
Problem 3. Consider the single-machine infinite-bus system shown in Fig. 2. The generator is described by the classical model, with $X_d' = 1.0 \text{ p.u.}$, and delivers steady-state power $P_G = 0.5 \text{ p.u.}$ to an infinite bus through a transmission line with reactance $X_L = 0.4 \text{ p.u.}$ Assume that $|E| = 1.8 \text{ p.u.}$, $V_\infty = 1\angle0^\circ \text{ p.u.}$, $H = 5 \text{ sec}$,

\[ E = |E|\angle\delta \]
\[ jX_d' \]
\[ jX_L \]
\[ V_\infty = 1\angle0^\circ \]

Figure 2: Single-machine infinite-bus system for Problem 3.

1. Find the two possible steady-state (equilibrium) values of power angle $\delta$ that lie in the interval $[0, 2\pi]$.
2. Which of these two equilibrium values are we likely to observe in practice? Hint: Linearize around the equilibrium value and consider the natural frequencies found from linear differential equation in $\Delta\delta$.

Problem 4. Suppose that in Problem 3 at $t = 0$, the line circuit breakers open. $P_M^0 = 0.5 \text{ p.u.}$ remains constant. Calculate $\delta$ and $\dot{\delta}$ at the end of 1 sec under two assumptions and compare:

(a) $D = 0$

(b) $D = 0.001$

Problem 5. Refer to Fig. 3 and assume that $P_M = 1.0 \text{ p.u.}$, $|E| = 1.5 \text{ p.u.}$, and $X_d' = 0.9 \text{ p.u.}$

\[ E = |E|\angle\delta \]
\[ jX_d' \]
\[ j0.5 \]
\[ j0.125 \]
\[ V_\infty = 1\angle0^\circ \]

Figure 3: System for Problem 5.

(a) At $t = 0$ the circuit breakers open and remain open. Determine if the transient is stable.

(b) Repeat (a) for $X_d' = 0.6 \text{ p.u.}$