LECTURE 1

• READING: Chapters 1 & 2 of SG&O

• THE CENTRAL PROBLEM IN POWER SYSTEMS

• REVIEW OF PHASORS

• POWER: INSTANTENOUS, AVERAGE & COMPLEX

THE CENTRAL PROBLEM IN POWER SYSTEMS

• It is to compute all electric variables given load consumptions. In a nutshell with a simple DC system

\[
V_S = 10V \\
\text{LOAD} \\
P = 10W, \text{ What are the values of } V \& I?
\]

\[
\text{Power:} \\
V \cdot I = P = 10 \\
V_S = 0.1 \cdot I + V \\
\Rightarrow I = \frac{10 - V}{0.1} \\
\Rightarrow 10 = V \cdot (\frac{10 - V}{0.1}) \\
\Rightarrow 1 = 10V - V^2 = 0 \\
\Rightarrow V^2 - 10V + 1 = 0
\]

\[
V = \frac{10 \pm \sqrt{10^2 - 4}}{2} = \left \{ \begin{array}{l}
\frac{10 + \sqrt{96}}{2} = 9.899 \Rightarrow I = 1.01A \\
\frac{10 - \sqrt{96}}{2} = 0.101 \Rightarrow I = 98.99A
\end{array} \right.
\]

Both solutions are possible in real life.

- The first one represents a case when the system is working properly (good voltage on the load, small currents)
- The second case is typical of a situation where there could be a fault (a short-circuit). We are interested in understanding this case to size properly conductors and to set protections properly.

REVIEW OF PHASORS

- The systems we will study extensively in this class are AC electric power systems.
- The use of phasors is very convenient to formulate and solve problems of interest.

- Let's go back to the DC example.

(I) \[ V = \frac{I}{R_1 + R_2} \]  
DC system

\[ V_{eq} = (R_1 + R_2)I \rightarrow \text{this is an algebraic expression, which we know how to manipulate.} \]

(II) \[ V(t) = V_m \cos(\omega t + \theta) \]

Kirchoff's laws always apply, i.e., the physics holds, so we can write the following differential equation

\[ V(t) = R.i(t) + L.\frac{di}{dt} \]

Q: Can we use some trick to represent (II) in such a way that we can use algebraic representation instead of differential equation? Hopefully, this will make our lives easier.

A: YES! PHASORS!!
Q: How do we go from $V(t)$, $L$, $R$ to $\mathcal{V}, \mathcal{Z}_L, \mathcal{Z}_R$?

\[ V(t) = L \frac{di}{dt} + R \cdot i \]

In steady-state: $i(t) = I_m \cdot \cos (\omega t + \Theta_i)$

\[ V_m \cos (\omega t + \Theta_V) = -L \cdot I_m \omega \sin (\omega t + \Theta_i) \]

But $-\sin (\omega t + \Theta_i) = \cos (\Theta_i + \frac{\pi}{2}) + \omega t$

\[ V_m \cos (\omega t + \Theta_V) = L \cdot I_m \omega \cdot \cos (\omega t + \Theta_i + \frac{\pi}{2}) \]

\[ I_m = \frac{V_m}{\omega L} \quad \Theta_i = \Theta_V - \frac{\pi}{2} \]

Now let's add and subtract $j V_m \sin (\omega t + \Theta_V)$ and $j L I_m \omega \cos (\omega t + \Theta_i + \frac{\pi}{2})$:

\[ V_m \cdot \cos (\omega t + \Theta_V) + j \sin (\omega t + \Theta_V) = L \cdot I_m \omega \cos (\omega t + \Theta_i + \frac{\pi}{2}) + j L I_m \omega \sin (\omega t + \Theta_i + \frac{\pi}{2}) \]
Euler's expansion:

\[ e^{j\theta} = \cos\theta + j\sin\theta \]

Then:

\[ V_m \ e^{j\theta_v} \ e^{j\omega t} = L \cdot W \cdot I_m \ e^{j(\theta_i + \omega t + \pi/2)} \]

\[ V_m \ e^{j\theta_v} \ e^{j\omega t} = L \cdot W \cdot I_m \ e^{j(\theta_i + \pi/2)} \ e^{j\omega t} \]

Let's define:

\[ \bar{V} = \frac{V_m}{\sqrt{2}} \ e^{j\theta_v} = L \cdot W \cdot e^{j\theta_i/2} \cdot I_m \ e^{j\omega t} \]

\[ \bar{I}_m = \frac{I_m}{\sqrt{2}} \ e^{j\theta_i} \]

\[ \bar{Z}_L = j \omega L \]

So we get:

\[ \bar{V} = \bar{Z}_L \cdot \bar{I} ! \]

Similarly, we can get:

\[ C \rightarrow \frac{1}{j\omega C} \]

\[ R \rightarrow \frac{1}{R} \]

**INSTANTANEOUS POWER**

\[ i(t) \]

\[ V(t) \]

\[ P(t) = V(t) \cdot i(t) \]

\[ V(t) = V_m \cos(\omega t + \theta_v) \]

\[ i(t) = I_m \cos(\omega t + \theta_i) \]

\[ P(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i) \]
Reminder: \( \cos(\theta_1) \cdot \cos \theta_2 = \frac{1}{2} [\cos(\theta_1 - \theta_2) + \cos(\theta_1 + \theta_2)] \)

Thus:

\[ P(t) = \frac{1}{2} V_m I_m \cdot [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)] \]

\[ P(t) = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(2\omega t + \theta_v + \theta_i) \]

For \( \theta_v - \theta_i = 0 \)

**Average Power**

\[ P = \frac{1}{T} \int_0^T \frac{V I}{2} \left[ \cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i) \right] dt = \]

\[ = \frac{V I}{2} \cos(\theta_v - \theta_i) + \frac{1}{T} \int_0^T \cos(2\omega t + \theta_v + \theta_i) dt = \]

\[ = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{1}{2\omega T} \sin(2\omega t + \theta_v + \theta_i) \bigg|_0^T \]

\[ P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \]

**Power Factor**

\[ PF = \cos(\theta_v - \theta_i) \]
ROOT-MEAN SQUARE

\[ V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T V^2(t) \, dt} = \frac{V_m}{\sqrt{2}} \]

Instead of \( \bar{V} = \frac{V_m}{\sqrt{2}} < \Theta_v \), we can define \( \bar{V} \) as: \( \bar{V} = V_{\text{rms}} < \Theta_v \).

Average power then becomes:

\[ P = V_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos(\Theta_v - \Theta_i) \]

From now onwards, we drop \( \text{Rms} \) and just use \( V \) to represent the RMS value of \( V(t) = V_m \cos(\omega t + \Theta_v) \).

COMPLEX POWER

Let's do a similar calculation, but instead of using \( V(t) \) and \( i(t) \), let's use \( \bar{V} \) and \( \bar{I} \):

\[ S = \bar{V} \bar{I}^* = V e^{j\Theta_v} I e^{-j\Theta_i} = V I e^{j(\Theta_v - \Theta_i)} \]

\[ P = \Re \left[ V I \left[ \cos(\Theta_v - \Theta_i) + j \sin(\Theta_v - \Theta_i) \right] \right] \]

\[ Q = V I \sin(\Theta_v - \Theta_i) \quad \text{→ Reactive power} \]

→ doesn’t do anything for us but it is extremely important for maintaining voltage.