In-class quiz: Thursday, September 6, 2018
Reading: Chapters 1 and 2 of GS&O

Problem 1. With $|V| = 100$ V, the instantaneous power $p(t)$ into a network $N$ has a maximum value 1707 W and a minimum value of -293 W.

1. Find a possible series $RL$ circuit equivalent to $N$.

Define $v(t)$ and $i(t)$ of the network $N$ as

$$v(t) = \sqrt{2}V \cos(\omega t + \theta_V),$$
$$i(t) = \sqrt{2}I \cos(\omega t + \theta_I).$$

Then, the instantaneous power into $N$ is

$$p(t) = v(t)i(t) = 2VI \cos(\omega t + \theta_V) \cos(\omega t + \theta_I) = VI \left[\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)\right].$$

The maximum value of $p(t)$ occurs when $\cos(2\omega t + \theta_V + \theta_I) = 1$ and, similarly, the minimum value of $p(t)$ occurs when $\cos(2\omega t + \theta_V + \theta_I) = -1$. So we have the following 2 equations:

$$P_{\text{max}} = VI \cos(\theta_V - \theta_I) + VI = 1707 \text{ W}$$
$$P_{\text{min}} = VI \cos(\theta_V - \theta_I) - VI = -293 \text{ W}$$

Subtracting one from the other, we get

$$2VI = 1707 + 293 \implies VI = 1000 \text{ W} \implies I = 10 \text{ A},$$

and adding the two and substituting $VI = 1000$ in, we get

$$2VI \cos(\theta_V - \theta_I) = 2000 \cos(\theta_V - \theta_I) = 1414 \implies \theta_V - \theta_I = \cos^{-1} \left( \frac{1414}{2000} \right) = \pm 45^\circ.$$ 

An inductive load causes current to lag the voltage, so we get $\theta_V - \theta_I = 45^\circ$. Now we use $V = 100\angle \theta_V$ and $I = 10\angle \theta_I$ to get

$$Z = \frac{V}{I} = \frac{100\angle \theta_V}{10\angle \theta_I} = 10\angle(\theta_V - \theta_I) = 10\angle 45^\circ \text{ \Omega}.$$

Finally we obtain $R = \Re(Z) = 7.07 \text{ \Omega}$ and $\omega L = \Im(Z) = 7.07 \Omega \implies L = 0.0188 \text{ H}.$
2. Find \( S = P + jQ \) into \( N \).

Using the voltage and current phasors from above, we get

\[
S = VI^* = (100\angle \theta_V)(10\angle \theta_I)^* = (100\angle \theta_V)(10\angle (-\theta_I)) = 1000\angle (\theta_V - \theta_I) = 1000\angle 45^\circ.
\]

So \( S = P + jQ \), where \( P = \Re(S) = 707 \text{ W} \) and \( Q = \Im(S) = 707 \text{ Var} \).

3. Find the maximum instantaneous power into \( L \) and compare with \( Q \).

The instantaneous power into \( L \) is

\[
p_L(t) = v_L(t)i(t) = \frac{di}{dt}i(t) = -L\sqrt{2}\omega I \sin(\omega t + \theta_I)\sqrt{2}I \cos(\omega t + \theta_I)
\]

\[
= 2\omega LI^2 \sin(\omega t + \theta_I) \cos(\omega t + \theta_I)
\]

\[
= \omega LI^2 \sin(2\omega t + 2\theta_I).
\]

The maximum value of \( p_L(t) \) occurs when \( \sin(2\omega t + 2\theta_I) = 1 \), and at this point,

\[
p_{L,max} = \omega LI^2 = 2\pi 60(0.0188)(10^2) = 707 \text{ W},
\]

which is equal to \( Q \).

**Problem 2.** A certain 1\( \phi \) load draws 5 MW at 0.7 power factor lagging. Determine the reactive power required from a parallel capacitor to bring the power factor of the parallel combination up to 0.9.

With the current power factor of 0.7 lagging, we solve the following for the current \( Q \):

\[
\tan(\cos^{-1}(0.7)) = \frac{Q_{\text{cur}}}{P} = \frac{Q_{\text{cur}}}{5} \implies Q_{\text{cur}} = 5.101 \text{ MVar}
\]

To reach a power factor of 0.9 lagging, we solve the following for the desired \( Q \):

\[
\tan(\cos^{-1}(0.9)) = \frac{Q_{\text{des}}}{P} = \frac{Q_{\text{des}}}{5} \implies Q_{\text{des}} = 2.422 \text{ MVar}
\]

Therefore, the reactive power required from a parallel capacitor to bring the power factor to 0.9 is

\[
Q_{\text{cap}} = Q_{\text{cur}} - Q_{\text{des}} = 5.101 - 2.422 = 2.679 \text{ MVar}
\]

**Problem 3.** A 3\( \phi \) load draws 200 kW at a PF of 0.707 lagging from a 440-V line. In parallel is a 3\( \phi \) capacitor bank that supplies 50 kVAr. Find the resultant power factor and current (magnitude) into the parallel combination.

In each phase, the load draws 200/3 kW at a PF of 0.707 lagging. So we solve for the reactive power that the load draws in each phase as follows:

\[
\tan(\cos^{-1}(0.707)) = \frac{Q_{\text{load,1}\phi}}{P_{\text{load,1}\phi}} = \frac{Q_{\text{load,1}\phi}}{200/3} \implies Q_{\text{load,1}\phi} = 66.69 \text{ kVar}.
\]

With the capacitor bank in parallel, the combined reactive power drawn becomes

\[
Q_{\text{combo,1}\phi} = Q_{\text{load,1}\phi} - Q_{\text{cap,1}\phi} = 66.69 - 50/3 = 50.02 \text{ kVar}.
\]
So the power factor of the combination is

\[
\cos \left( \tan^{-1} \left( \frac{50.02}{200/3} \right) \right) = 0.7999 \approx 0.8 \text{ lagging}
\]

The current magnitude into the combination is

\[
|I_{\text{combo},1\phi}| = \frac{|S_{\text{combo},1\phi}|}{|V|} = \frac{\sqrt{P_{1\phi}^2 + Q_{\text{combo},1\phi}^2}}{|V|} = \frac{\sqrt{(200/3)^2 + 50.02^2}}{440} = 189 \text{ A, per phase}
\]

**Problem 4.** A 1\(\phi\) load draws 10 kW from a 416-V line at a power factor of 0.9 lagging.

1. Find \(S = P + jQ\).

At power factor 0.9 lagging, the complex power drawn is solved as

\[
\tan (\cos^{-1}(0.9)) = \frac{Q}{P} = \frac{Q}{P_{\text{load}}} = \frac{Q_{\text{load}}}{10} \implies Q = 4.84 \text{ kVar}.
\]

Then, \(S = 10 + j4.84 \text{ kVA}\).

2. Find \(|I|\).

\[
|I| = \frac{|S|}{|V|} = \frac{\sqrt{10^2 + 4.84^2} \times 1000}{416} = 26.7 \text{ A}
\]

3. Assume that \(\angle I = 0\) and find the instantaneous power \(p(t)\).

\[
p(t) = v(t)i(t) = \sqrt{2}V \cos(\omega t + \theta_V) \sqrt{2}I \cos(\omega t) = 2VI \cos(\omega t + \theta_V) \cos(\omega t) = VI \cos \theta_V + VI \cos(2\omega t + \theta_V)
\]

\[
= P + VI \left[ \cos(2\omega t) \cos \theta_V - \sin(2\omega t) \sin \theta_V \right] = P + VI \cos \theta_V \cos(2\omega t) - VI \sin \theta_V \sin(2\omega t)
\]

\[
= P + P \cos(2\omega t) - Q \sin(2\omega t) = P(1 + \cos(2\omega t)) - Q \sin(2\omega t)
\]

\[
= 10(1 + \cos(2\omega t)) - 4.84 \sin(2\omega t) \text{ kW}
\]

**Problem 5.** A small manufacturing plant is located 2km down a transmission line, which has a series reactance of 0.5 \(\Omega/\text{km}\). The line resistance is negligible. The line voltage plant is 480\(\angle 0^\circ\) V (rms), and the plant consumes 120 kW at 0.85 power factor lagging. Determine the voltage and power factor at the sending end of the transmission line by using:

1. A complex power approach.

The load draws 120 kW at 0.85 power factor lagging. We solve for the reactive power drawn by the load as

\[
\tan (\cos^{-1}(0.85)) = \frac{Q_{\text{load}}}{P_{\text{load}}} = \frac{Q_{\text{load}}}{120} \implies Q_{\text{load}} = 74.37 \text{ kVar}.
\]

Therefore, the complex power drawn by the load is \(S_{\text{load}} = 120 + j74.37 \text{ kVA}\). We can now solve for the current into the load as

\[
I = \left( \frac{S_{\text{load}}}{V_{\text{load}}} \right)^* = \left( \frac{120 + j74.37}{480\angle 0^\circ} \right)^* = 294.1\angle(-31.79^\circ) \text{ A}.
\]
The loss in the line can be computed as

\[ S_{\text{line}} = V_{\text{line}} I_{\text{line}}^* = Z_{\text{line}} I_{\text{load}} I^* = j2(0.5)294.1^2 = j86.51 \text{ kVA}. \]

Thus, the complex power supplied by the source is

\[ S_{\text{source}} = S_{\text{load}} + S_{\text{line}} = 120 + j74.37 + j86.51 = 120 + j160.88 \text{ kVA} = 200.7\angle53.28^\circ \text{ kVA}. \]

So the power factor at the sending end is \( \cos(53.28^\circ) = 0.598 \), lagging.

Finally, the voltage at the sending end is

\[ V_{\text{source}} = \frac{S_{\text{source}}}{I^*} = \frac{200.7\angle53.28^\circ}{294.1\angle31.79^\circ} = 682.4\angle21.5^\circ \text{ V}. \]

2. A circuit analysis approach.

Using KVL, we have

\[ V_{\text{source}} = Z_{\text{line}} I + V_{\text{load}} = j1(294.1\angle(-31.79^\circ)) + 480 = 682.4\angle21.5^\circ \text{ V}. \]

And the power factor is \( \cos(\theta_V - \theta_I) = \cos(21.5^\circ + -31.79^\circ) = 0.598 \), lagging.