**ECE 476 – Power System Analysis Fall 2018**  
**Homework 4**

**In-class quiz:** Tuesday September 27, 2018

**Problem 1.** A 500-km, 500-kV, 60-Hz uncompensated three-phase line has a positive-sequence series impedance \( \bar{z} = 0.03 + j0.35 \) \( \Omega/\text{km} \) and a positive-sequence shunt admittance \( \bar{y} = j4.4 \times 10^{-6} \) \( \text{S/km} \). Calculate:

(a) \( \bar{Z}_c \)

\[
\bar{Z}_c = \sqrt{\frac{\bar{z}}{\bar{y}}} = \sqrt{\frac{0.03 + j0.35}{j4.4 \times 10^{-6}}} = \sqrt{79837 - 4.899^2} = 282.6 \angle -2.45^\circ = 282.3 - j12.08 \ \Omega
\]

(b) \( (\bar{\gamma}_d) \)

\[
\bar{\gamma}_d = \sqrt{\bar{z} \bar{y}} = 500 \sqrt{(0.03 + j0.35)(j4.4 \times 10^{-6})} = 500 \sqrt{1.546 \times 10^{-6} \angle 175.1^\circ} = 0.6216 \angle 87.55^\circ = 0.02657 + j0.6210 \ \text{per unit}
\]

(c) The exact ABCD parameters for this line.

\[
e^{\bar{\gamma}_d} = e^{0.02657e^{j0.6210}} = 1.027\angle0.6210 \ \text{rad} = 0.8352 + j0.5975
e^{-\bar{\gamma}_d} = e^{-0.02657e^{-j0.6210}} = 0.9738\angle -0.6210 \ \text{rad} = 0.7920 - j0.5666
\]

\[
\cosh \bar{\gamma}_d = \frac{e^{\bar{\gamma}_d} + e^{-\bar{\gamma}_d}}{2} = \frac{1.027\angle0.6210 + 0.9738\angle -0.6210}{2} = 0.8137\angle1.088^\circ
\]

\[
\sinh \bar{\gamma}_d = \frac{e^{\bar{\gamma}_d} - e^{-\bar{\gamma}_d}}{2} = \frac{1.027\angle0.6210 - 0.9738\angle -0.6210}{2} = 0.5825\angle87.87^\circ
\]

\[
A = D = \cosh \bar{\gamma}_d = 0.8137\angle1.088^\circ \ \text{per unit}
\]

\[
B = \bar{Z}_c \sinh \bar{\gamma}_d = (282.6\angle -2.45^\circ)(0.5825\angle87.87^\circ) = 164.6\angle85.42^\circ \ \Omega
\]

\[
C = \frac{1}{\bar{Z}_c} \sinh \bar{\gamma}_d = \frac{0.5825\angle87.87^\circ}{282.6\angle -2.45^\circ} = 2.061 \times 10^{-3} \angle 90.32^\circ \ \text{S}
\]

**Problem 2.** A 320-km 500-kV, 60-Hz three-phase uncompensated line has a positive-sequence series reactance \( x = 0.34 \) \( \Omega/\text{km} \) and a positive-sequence shunt admittance \( y = 4.5 \times 10^{-6} \) \( \text{S/km} \). Neglecting losses, calculate:

(a) Its characteristic impedance \( Z_c \).

\[
\bar{Z}_c = \sqrt{\frac{\bar{z}}{\bar{y}}} = \sqrt{\frac{j0.34}{j4.5 \times 10^{-6}}} = 274.9 \ \Omega
\]

(b) The value of \( \bar{\gamma}_d \).

\[
\bar{\gamma}_d = \sqrt{\bar{z} \bar{y}} = 320 \sqrt{(j0.34)(j4.5 \times 10^{-6})} = j0.3958 \ \text{per unit}
\]
(c) The exact ABCD parameters for this line.
\[
\begin{align*}
\bar{e}^{\gamma d} &= e^{j0.3958} = 1\angle 22.68^\circ = 0.9227 + j0.3855 \\
e^{-\gamma d} &= e^{-j0.3958} = 1\angle -22.68^\circ = 0.9227 - j0.3855 \\
cosh \gamma d &= \frac{e^{\gamma d} + e^{-\gamma d}}{2} = \frac{0.9227 + j0.3855 + 0.9227 - j0.3855}{2} = 0.9227 \\
\sinh \gamma d &= \frac{e^{\gamma d} - e^{-\gamma d}}{2} = \frac{0.9227 + j0.3855 - 0.9227 + j0.3855}{2} = j0.3855 \\
A &= D = \cosh \gamma d = 0.9227 \text{ per unit} \\
B &= Z_c \sinh \gamma d = (274.9)(j0.3855) = j105.97 \Omega \\
C &= \frac{1}{Z_c} \sinh \gamma d = \frac{j0.3855}{274.9} = j1.402 \times 10^{-3} \text{ S}
\end{align*}
\]

(d) The surge impedance loading in MW.
\[
SIL = \frac{V_{\text{rated,lt}}^2}{Z_c} = \frac{500^2}{274.9} = 909.4 \text{ MW, } 3\phi
\]

**Problem 3.** The per-phase impedance of a short three-phase transmission line is 0.5\(\angle 53.15^\circ\) \(\Omega\). The three-phase load at the receiving end is 900 kW at 0.8 p.f. lagging. If the line-to-line sending-end voltage is 3.3 kV, determine:

(a) The receiving-end line-to-line voltage in kV.
\[
\bar{S}_L = \frac{300000}{0.8} \angle \cos^{-1} 0.8 = 300000 + j225000 \text{ VA, and } \bar{V}_S = \frac{3300}{\sqrt{3}} \angle 0^\circ = 1905\angle 0^\circ \text{ V}
\]
Let the receiving-end line-to-neutral voltage be \(V_R\angle \theta\). Then,
\[
\bar{S}_L = (V_R\angle \theta)\bar{I}^* = (V_R\angle \theta) \left( \frac{\bar{V}_S - V_R\angle \theta}{Z} \right)^* \\
\bar{S}_L Z^* = 1.905V_R\angle \theta - V_R^2 \\
(300000 + j225000)(0.5\angle 53.15^\circ)^* = 1905V_R\angle \theta - V_R^2 \\
179982 - j52562.5 = 1905V_R \cos \theta + j1905V_R \sin \theta - V_R^2.
\]
Equating the real and imaginary parts, we obtain
\[
179982 + V_R^2 = 1905V_R \cos \theta \\
-52562.5 = 1905V_R \sin \theta.
\]
Taking the square of each and summing, we get
\[
(179982 + V_R^2)^2 + (-52562.5)^2 = 1905^2V_R^2 \cos^2 \theta + 1905^2V_R^2 \sin^2 \theta \\
3.239352 \times 10^{10} + 359964V_R^2 + V_R^4 + 2.76282 \times 10^9 = 3629025V_R^2 \\
V_R^4 - 3269061V_R^2 + 3.515634 \times 10^{10} = 0 \\
V_R^2 = \frac{3269061 \pm \sqrt{3269061^2 - 4(3.515634 \times 10^{10})}}{2} \\
V_R^2 = 3258271 \text{ or } 10790
\]
Therefore,
\[
V_R = \pm 1805 \text{ V or } \pm 103.9 \text{ V.}
\]
Eliminating the negative voltages and the smaller root, we get \(V_R = 1.805 \text{ kV}\). Solving for \(\theta\) from an earlier equation, we get \(\theta = -0.8759^\circ\). The line-to-line receiving-end voltage is \(\sqrt{3}(1.805) = 3.126 \text{ kV}\).
(b) The line current.

\[ \bar{S}_L = \bar{V}_R \bar{I}^* \]

\[ 300000 + j225000 = (1805\angle -0.8759^\circ)\bar{I}^* \]

\[ \bar{I} = 207.8\angle -37.75^\circ \text{ A} \]

(c) The phasor diagram with the line current \( I \), as reference.

[Diagram of phasor diagram]

**Problem 4.** To maintain a safe “margin” of stability, system designers have decided that the power angle \( \theta_{12} := \theta_1 - \theta_2 \), where \( \theta_1 \) is the phase angle of the sending-end voltage and \( \theta_2 \) is the phase angle of the receiving-end voltage, cannot be greater than 45°. We wish to transmit 500 MW though a 300-mile line and need to pick a transmission-line voltage level. Consider 138−, 345−, and 765−kV lines. Which voltage level(s) would be suitable? As a first approximation, assume that the voltage magnitudes on sending and receiving ends are equal, i.e., \( V_1 = V_2 \) and the lines are lossless, i.e., \( \bar{\gamma} = j\beta \), with \( \beta = 0.002 \text{ rad/mi.} \)

The real power delivered to the receiving end for a lossless line is

\[ P = \frac{V_R V_S}{X'} \sin(\theta_1 - \theta_2). \]

Next, we determine \( X' \) for a lossless line as follows:

\[ X' = \omega L \left( \frac{\sin(\beta l)}{\beta l} \right) = 2\pi 60(300) L \left( \frac{\sin(0.002 \cdot 300)}{0.002 \cdot 300} \right) = 106432.6 L. \]

We are given typical values for \( C \) for the three voltage levels, so we can solve for \( L \) using \( \beta = \omega \sqrt{LC} \).

138 kV: \( C_{138} = 8.84 \times 10^{-12} \text{ F/m} = 1.422 \times 10^{-8} \text{ F/mi} \quad \Rightarrow \quad L_{138} = 1.979 \times 10^{-3} \text{ H/mi} \quad \Rightarrow \quad X'_{138} = 210.63 \text{ Ω} \)

345 kV: \( C_{345} = 11.59 \times 10^{-12} \text{ F/m} = 1.865 \times 10^{-8} \text{ F/mi} \quad \Rightarrow \quad L_{345} = 1.509 \times 10^{-3} \text{ H/mi} \quad \Rightarrow \quad X'_{345} = 160.6 \text{ Ω} \)

765 kV: \( C_{765} = 12.78 \times 10^{-12} \text{ F/m} = 2.056 \times 10^{-8} \text{ F/mi} \quad \Rightarrow \quad L_{765} = 1.369 \times 10^{-3} \text{ H/mi} \quad \Rightarrow \quad X'_{765} = 145.7 \text{ Ω} \)

Assuming \( V_R = V_S = V \), we get

\[ P = \frac{V^2}{X'} \sin(\theta_1 - \theta_2). \]

For the case of 138 kV line,

\[ \sin(\theta_1 - \theta_2) = \frac{PX'_{138}}{V_{138}^2} = \frac{(500 \times 10^6)(210.63)}{138000^2} = 5.53, \]

which does not have a solution. For the case of 345 kV line,

\[ \sin(\theta_1 - \theta_2) = \frac{PX'_{345}}{V_{345}^2} = \frac{(500 \times 10^6)(160.6)}{3450000^2} = 0.6746 \quad \Rightarrow \quad \theta_1 - \theta_2 = 42.43^\circ < 45^\circ, \]
which satisfies the safe margin of stability. For the case of 765 kV line,

\[ \sin(\theta_1 - \theta_2) = \frac{P_{X'_{765}}}{V_{Y_{765}}^2} = \frac{(500 \times 10^6)(145.7)}{765000^2} = 0.1245 \implies \theta_1 - \theta_2 = 7.15^\circ < 45^\circ, \]

which is well under the safe margin of stability. Therefore, both the 345 kV and 765 kV line are suitable to transmit 500 MW through the line.

**Problem 5.** Given a transmission line described by a total series impedance \( Z = \bar{z}d = 20 + j80 \) \( \Omega \) and a total shunt admittance \( Y = \bar{y}d = j5 \times 10^{-4} \) \( \Omega \).

(a) Find its characteristic impedance \( \bar{Z}_c \), \( e^{\gamma d} \), \( \sinh \gamma d \), and \( \cosh \gamma d \).

\( \bar{Z}_c = \sqrt{\frac{\bar{z}}{\bar{y}}} = \sqrt{\frac{\bar{z}d}{\bar{y}d}} = \sqrt{\frac{20 + j80}{j5 \times 10^{-4}}} = \sqrt{164924.225 \angle -14.036^\circ} = 406.11 \angle -7.02^\circ \) \( \Omega \)

\( \gamma d = \sqrt{\bar{z}y} = \sqrt{(20 + j80)(j5 \times 10^{-4})} = \sqrt{0.04123 \times 165.96^\circ} = 0.2031 \angle 82.98^\circ = 0.02482 + j0.2016 \) per unit

\[ e^{\gamma d} = e^{0.02482 e^{j0.2016}} = 1.025 \angle 0.2016 \text{ rad} = 1.0042 + j0.2052 \]

\[ e^{-\gamma d} = e^{-0.02482 e^{-j0.2016}} = 0.9755 \angle -0.2016 \text{ rad} = 0.9557 - j0.1953 \]

\[ \cosh \gamma d = \frac{e^{\gamma d} + e^{-\gamma d}}{2} = \frac{1.0042 + j0.2052 + 0.9557 - j0.1953}{2} = 0.9799 \angle 0.2885^\circ \]

\[ \sinh \gamma d = \frac{e^{\gamma d} - e^{-\gamma d}}{2} = \frac{1.0042 + j0.2052 - 0.9557 + j0.1953}{2} = 0.2017 \angle 83.10^\circ \]

(b) Suppose that the line is terminated in its characteristic impedance \( \bar{Z}_c \). Find the efficiency of the transmission line in this case, i.e., find \( \eta = -P_{21}/P_{12} \), where \( P_{21} \) is the active power flowing from the receiving end to the sending end of the line, and \( P_{12} \) is the active power flowing from the sending end to the receiving end of the line.

\[ \bar{V}_1 = \bar{V}_2 \cosh \gamma d + \bar{I}_2 \bar{Z}_c \sinh \gamma d \]

\[ \bar{I}_1 = \frac{\bar{V}_2}{\bar{Z}_c} \sinh \gamma d + \bar{I}_2 \cosh \gamma d \]

If the line is terminated in \( \bar{Z}_c \), then \( \bar{V}_2 = \bar{Z}_c \bar{I}_2 \) and the above equations become

\[ \bar{V}_1 = \bar{V}_2 \cosh \gamma d + \bar{V}_2 \sinh \gamma d = \bar{V}_2 (\cosh \gamma d + \sinh \gamma d) = \bar{V}_2 e^{\gamma d} \]

\[ \bar{I}_1 = \bar{I}_2 \sinh \gamma d + \bar{I}_2 \cosh \gamma d = \bar{I}_2 (\cosh \gamma d + \sinh \gamma d) = \bar{I}_2 e^{\gamma d}. \]

Thus,

\[ \bar{V}_2 = \bar{V}_1 e^{-\gamma d} \text{ and } \bar{I}_2 = \bar{I}_1 e^{-\gamma d}. \]

With \( \gamma = \alpha + j\beta \), the complex power from receiving end to sending end can be written as

\[ S_{21} = -\bar{V}_2 \bar{I}_2^* = -\bar{V}_1 e^{-\gamma d} (\bar{I}_1 e^{-\gamma d})^* = -\bar{V}_1 e^{-\alpha d} e^{-j\beta d} \bar{I}_1^* e^{-\alpha d} e^{j\beta d} = -S_{12} e^{-2\alpha d}. \]

Since \( \alpha \) is real, the efficiency of the line is

\[ \eta = -\frac{P_{21}}{P_{12}} = e^{-2\alpha d} = e^{-2(0.02482)} = 0.952 \]