Problem 1. Compute the elements of the third row of $Y_{bus}$ for the power system in Example 6.9 of textbook.

Solution. We find each entry in the third row of the admittance matrix as follows:

\[ Y_{31} = Y_{32} = Y_{35} = 0. \]

\[ Y_{34} = \frac{-1}{R_{34} + jX_{34}} = \frac{-1}{0.00075 + j0.01} = -7.458 + j99.44 \text{ p.u.} \]

\[ Y_{33} = \frac{1}{R_{34} + jX_{34}} + j \frac{B_{34}'}{2} = \frac{1}{0.00075 + j0.01} + j \frac{0}{2} = 7.458 - j99.44 \text{ p.u.} \]

So the third row of the admittance matrix is

\[ Y_3 = \begin{bmatrix} 0 & 0 & 7.458 - j99.44 & -7.458 + j99.44 & 0 \end{bmatrix}. \]

Problem 2. Given the impedance diagram of a simple system as shown in Figure 1, draw the admittance diagram for the system and develop the 4 x 4 bus admittance matrix $Y_{bus}$ by inspection.

Solution. The admittance diagram is shown in Figure 2.
The structure of the admittance matrix is given as
\[
\bar{Y}_{\text{bus}} = \begin{bmatrix}
\bar{Y}_{11} & \bar{Y}_{12} & \bar{Y}_{13} & \bar{Y}_{14} \\
\bar{Y}_{21} & \bar{Y}_{22} & \bar{Y}_{23} & \bar{Y}_{24} \\
\bar{Y}_{31} & \bar{Y}_{32} & \bar{Y}_{33} & \bar{Y}_{34} \\
\bar{Y}_{41} & \bar{Y}_{42} & \bar{Y}_{43} & \bar{Y}_{44} \\
\end{bmatrix},
\]

where the components are found as follows
\[
\bar{Y}_{11} = \bar{y}_{10} + \bar{y}_{12} + \bar{y}_{13}, \quad \bar{Y}_{22} = \bar{y}_{20} + \bar{y}_{12} + \bar{y}_{23}, \\
\bar{Y}_{33} = \bar{y}_{13} + \bar{y}_{23} + \bar{y}_{34}, \quad \bar{Y}_{44} = \bar{y}_{34}, \\
\bar{Y}_{12} = \bar{Y}_{21} = -\bar{y}_{12}, \quad \bar{Y}_{13} = \bar{Y}_{31} = -\bar{y}_{13}, \\
\bar{Y}_{23} = \bar{Y}_{32} = -\bar{y}_{23}, \quad \bar{Y}_{34} = \bar{Y}_{43} = -\bar{y}_{34}.
\]

Plugging in the values from the admittance diagram, the admittance matrix becomes
\[
\bar{Y}_{\text{bus}} = j\begin{bmatrix}
-8.5 & 2.5 & 5.0 & 0 \\
2.5 & -8.75 & 5.0 & 0 \\
5.0 & 5.0 & -22.5 & 12.5 \\
0 & 0 & 12.5 & -12.5 \\
\end{bmatrix}
\]

**Problem 3.** A load \( L \) consuming 1 p.u. of active power and 0.5 p.u. of reactive power is connected to a generator \( G_1 \) through a short transmission line with \( Z' = 0.02 + j0.06 \) p.u. Also, there is a capacitor connected to the load bus with admittance \( Y_{\text{cap}} = j0.25 \) p.u. The generator voltage is voltage \( V_{G1} = 1\angle0^\circ \).

**Solution**

a) The one line diagram is shown in Figure 3.

b) The admittance matrix can be found by inspection as
\[
\bar{Y} = \begin{bmatrix}
5 - j15 & -5 + j15 \\
-5 + j15 & 5 - j14.75
\end{bmatrix}.
\]

c) The power flow equations can now be written for each bus.

Bus 1:
\[
P_1 = V_1^2G_{11} + V_1V_2[G_{12}\cos(\theta_1 - \theta_2) + B_{12}\sin(\theta_1 - \theta_2)] = 5 + V_2[-5\cos(\theta_2) - 15\sin(\theta_2)]
\]
\[
Q_1 = -V_1^2B_{11} + V_1V_2[G_{12}\sin(\theta_1 - \theta_2) - B_{12}\cos(\theta_1 - \theta_2)] = 15 + V_2[5\sin(\theta_2) - 15\cos(\theta_2)]
\]
1\angle0^\circ \quad \text{Bus 1} \quad G1 \quad j0.25 \quad S = 1+j0.5 \quad \text{Bus 2}

**Figure 3: One Line Diagram**

Bus 2:

\[
P_2 = V_2^2 G_{22} + V_2 V_1 [G_{21} \cos(\theta_2 - \theta_1) + B_{21} \sin(\theta_2 - \theta_1)] = 5V_2^2 + V_2 [-5 \cos(\theta_2) + 15 \sin(\theta_2)] = -1
\]

\[
Q_2 = -V_2^2 B_{22} + V_2 V_1 [G_{21} \sin(\theta_2 - \theta_1) - B_{21} \cos(\theta_2 - \theta_1)] = 14.75V_2^2 + V_2 [-5 \sin(\theta_2) - 15 \cos(\theta_2)] = -0.5
\]

**Problem 4.** Solve the following equation by the Newton-Raphson method:

\[
2x_1^2 + x_2^2 - 8 = 0
\]

\[
x_1^2 - x_2^2 + x_1 x_2 - 4 = 0
\]

Start with an initial guess of \(x_1 = 1\) and \(x_2 = 1\).

**Solution.** Define \(x = [x_1, x_2]^\top\), and

\[
f = \begin{bmatrix}
2x_1^2 + x_2^2 - 8 \\
x_1^2 - x_2^2 + x_1 x_2 - 4 = 0
\end{bmatrix}.
\]

Then,

\[
\frac{\partial f}{\partial x} = \begin{bmatrix}
4x_1 & 2x_2 \\
2x_1 + x_2 & -2x_2 + x_1
\end{bmatrix}.
\]

The update rule using Newton-Raphson method is:

\[
x^{(k+1)} = x^{(k)} - \left( \frac{\partial f(x^{(k)})}{\partial x} \right)^{-1} f(x^{(k)})
\]

After 4 iterations, you will get the solution \(x = [1.8091, 1.2060]^\top\).

**Problem 5.** Assume a 1\(+j0.5\) per unit load at bus 2 is being supplied by a generator at bus 1 through a transmission line with series impedance of 0.05 \(+j0.1\) per unit. Assuming bus 1 is the swing bus with a fixed per unit voltage of 1.0\(\angle0^\circ\), use Newton-Raphson method to calculate the voltage at bus 2 after three iterations.

**Solution.** Similar to Problem 3, the admittance matrix can be found by inspection as

\[
\bar{Y} = \begin{bmatrix}
4 - j8 & -4 + j8 \\
-4 + j8 & 4 - j8
\end{bmatrix}.
\]

The power flow equations at bus 2 are

\[
P_2 = V_2^2 G_{22} + V_2 V_1 [G_{21} \cos(\theta_2 - \theta_1) + B_{21} \sin(\theta_2 - \theta_1)] = 4V_2^2 + V_2 [-4 \cos(\theta_2) + 8 \sin(\theta_2)] = -1
\]

\[
Q_2 = -V_2^2 B_{22} + V_2 V_1 [G_{21} \sin(\theta_2 - \theta_1) - B_{21} \cos(\theta_2 - \theta_1)] = 8V_2^2 + V_2 [-4 \sin(\theta_2) - 8 \cos(\theta_2)] = -0.5
\]
Define $x = [\theta_2, V_2]^\top$, and
\[
f = \begin{bmatrix} 4V_2^2 - 4V_2 \cos \theta_2 + 8V_2 \sin \theta_2 + 1 \\ 8V_2^2 - 4V_2 \sin \theta_2 - 8V_2 \cos \theta_2 + 0.5 \end{bmatrix}.
\]
Then,
\[
\frac{\partial f}{\partial x} = \begin{bmatrix} 4V_2 \sin \theta_2 + 8V_2 \cos \theta_2 & 8V_2 - 4 \cos \theta_2 + 8 \sin \theta_2 \\ -4V_2 \cos \theta_2 + 8V_2 \sin \theta_2 & 16V_2 - 4 \sin \theta_2 - 8 \cos \theta_2 \end{bmatrix}.
\]
The update rule using Newton-Raphson method is:
\[
x^{(k+1)} = x^{(k)} - \left( \frac{\partial f(x^{(k)})}{\partial x} \right)^{-1} f(x^{(k)})
\]
The iteration process is shown below:

\begin{align*}
x^{(0)} &= [0, 1]^\top \\
f(x^{(0)}) &= [1.0, 0.5]^\top \\
\frac{\partial f(x^{(0)})}{\partial x} &= \begin{bmatrix} 8 & 4 \\ -4 & 8 \end{bmatrix}
\end{align*}

\begin{align*}
x^{(1)} &= [-0.075, 0.9]^\top \\
f(x^{(1)}) &= [0.1106, 0.07]^\top \\
\frac{\partial f(x^{(1)})}{\partial x} &= \begin{bmatrix} 6.91 & 2.61 \\ -4.13 & 6.72 \end{bmatrix}
\end{align*}

\begin{align*}
x^{(2)} &= [-0.0848, 0.8836]^\top \\
f(x^{(2)}) &= [0.0025, 0.00175]^\top \\
\frac{\partial f(x^{(2)})}{\partial x} &= \begin{bmatrix} 6.74 & 2.41 \\ -4.12 & 6.50 \end{bmatrix}
\end{align*}

\begin{align*}
x^{(3)} &= [-0.085, 0.8832]^\top \\
f(x^{(3)}) &= [1.5 \times 10^{-6}, 1.09 \times 10^{-6}]^\top
\end{align*}