SMALL-SIGNAL STABILITY ANALYSIS

Consider a system comprised of a single generator connected to an infinite bus.

\[
\frac{dS}{dt} = w - w_0,
\]

\[
\frac{dw}{dt} = -\frac{D}{M} (w - w_0) + P_m - \frac{E V_{oo}}{X M} \sin S,
\]

where \( X = X_d + X_e \)

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**Equilibrium Points:**

\[
\frac{dS}{dt} = 0 \rightarrow \boxed{w_e = w_0}
\]

\[
\frac{dw}{dt} = 0 \rightarrow 0 = P_m - \frac{E V_{oo}}{X} \sin S
\]

\[
S_e = \arcsin \left( \frac{P_m}{E V_{oo}} \right) \quad \text{(two eq. points)}
\]
1st eq. point \[ W_e = W_0 \]
\[ S_e^{(1)} = \arcsin \left( \frac{P_m}{E V_{oo}} \right) \]

2nd eq. point \[ W_e = W_0 \]
\[ S_e^{(2)} = \pi - \arcsin \left( \frac{P_m}{E V_{oo}} \right) \]

Q. If the system is operating in one of the two equilibrium points and we slightly perturb the system, e.g., \( V_{oo} \rightarrow V_{oo} + \Delta V_{oo} \) for \( \Delta V_{oo} \) small, or \( P_m \rightarrow P_m + \Delta P_m \) for \( \Delta P_m \) small, temporarily:

What happens after the perturbation dissipates?
Does the system go back to operating in the pre-disturbance equilibrium point?

This question can be answered by linearizing the system around the equilibrium points and studying the stability properties of the linearization.

\[ \dot{x}_e = f(x_e, u) \quad \text{(before perturbation)} \]

\[ \text{(Nominal input : } P_m, V_{ac}) \]

\[ \frac{d}{dt}(x_e + \Delta x) = f(x_e + \Delta x, u + \Delta u) \quad \text{(When perturbation takes place).} \]

\[ f(x_e + \Delta x, u + \Delta u) \approx f(x_e) + \frac{\partial f}{\partial x}|_{x_e,u} \Delta x + \frac{\partial f}{\partial u}|_{x_e,u} \Delta u \]

\[ \text{Taylor series expansion} \]

But \( f(x_e) = 0 \) (the system is in equilibrium before the disturbance occurs) and \( \frac{d}{dt}(x_e + \Delta x) = \frac{d}{dt}x_e + \frac{d}{dt}\Delta x \). Thus

\[ \frac{d}{dt}\Delta x = \frac{\partial f}{\partial x}|_{x_e,u} \Delta x + \frac{\partial f}{\partial u}|_{x_e,u} \Delta u \]
\[
\frac{d\Delta X}{dt} = A \Delta X + B \Delta u
\]

This is a linear system. If the matrix \( A \) is Hurwitz, i.e., all eigenvalues have strictly negative real parts, then the system is small-signal stable, i.e., after the perturbation ceases, the system will go back to operating in its pre-disturbance equilibrium point.

In our case,
\[
X = [S, W]^T
\]

and
\[
\frac{d}{dt} \begin{bmatrix} \Delta S \\ \Delta W \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{E_{V0}}{XM^2} & -\frac{D}{M} \end{bmatrix} \begin{bmatrix} \Delta S \\ \Delta W \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & \frac{E_{sm0}}{XM} \end{bmatrix} \Delta V_n.
\]

We need to plug the two equilibrium points computed earlier and calculate the eigenvalues of \( A \).
Eigenvalues of $A$:

$$\begin{vmatrix}
\lambda & -1 \\
-\frac{E V_{\infty}}{X} \cos S_e & \lambda + \frac{D}{M}
\end{vmatrix} = 0$$

$$\lambda^2 + \frac{D}{M} \lambda + \frac{E V_{\infty}}{M X} \cos S_e = 0$$

$$\lambda = \frac{-\frac{D}{M} \pm \sqrt{\left(\frac{D}{M}\right)^2 - 4 \frac{E V_{\infty}}{M X} \cos S_e}}{2}$$

If $S_e < \frac{\pi}{2}$, then $\cos S_e > 0$

and $\sqrt{\left(\frac{D}{M}\right)^2 - 4 \frac{E V_{\infty}}{M X} \cos S} < \frac{D}{M}$;

thus both eigenvalues are real and strictly smaller than zero

$\Rightarrow$ the system is small-signal stable.

If $S_e > \frac{\pi}{2}$, then $\cos S_e < 0$

and one of the two eigenvalues will be greater than zero.
\[ S_e^{(1)} = \arcsin \left( \frac{P_m}{E V_{cc}} \right) < \frac{\pi}{2} \]

\[ S_e^{(2)} = \arcsin \left( \frac{P_m}{E V_{cc}} \right) > \frac{\pi}{2} \]

The system is small-signal stable around this equilibrium point.